Reduced Coulomb Energy Networks

- Intermediate method to Parzen window and k-nearest neighbor estimation
  - Parzen window uses fixed window size
  - K-nn uses variable window size: increase window size until enough samples are enclosed
- Adjust window size until you encounter points of a different category
- Can be implemented as a neural network
- Gets name from electrostatics
  - Energy associated with charged particles

Decision regions created by RCE network

Gray: Class1
Pink: Class 2
Red: Ambiguous

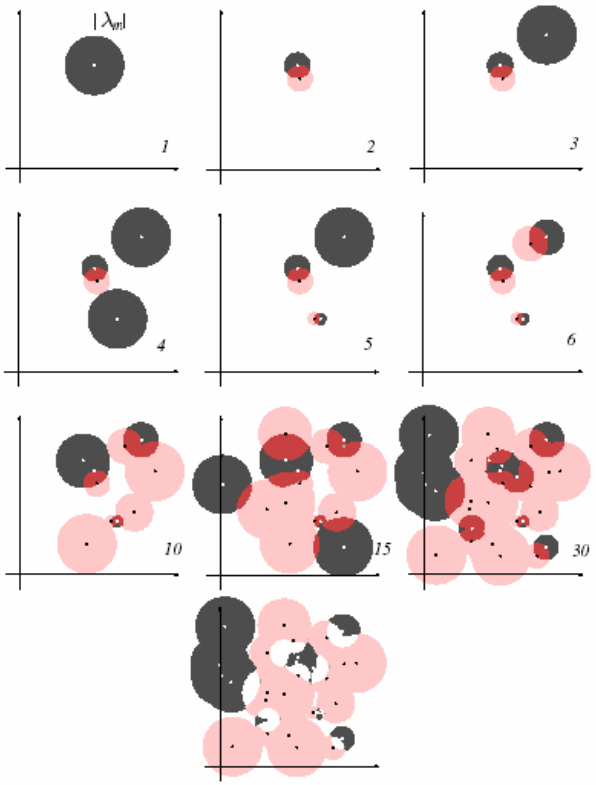

FIGURE 4.26. During training of an RCE network, each pattern has a parameter—equivalent to a radius in the d-dimensional space—that is adjusted to be as large as possible without enclosing any points from a different category (up to a maximum λ_m). As new patterns are presented, each such radius is decreased so that no sphere encloses a pattern of a different category. In this way, each sphere can enclose only patterns having the same category label. In this figure, the regions corresponding to one category are pink, and the other category are gray. Ambiguous regions (those enclosed by spheres of both categories) are shown in dark red. The number of points is shown in each component figure. The figure at the bottom shows the final decision regions, colored by category. From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001 by John Wiley & Sons, Inc.

Training Reduced Coulomb Energy Networks

- Adjust each radius to be as large as possible (upto a maximum) without containing point from another category
- For each training sample x_j, j=1,...,n set radius

Algorithm 4. (RCE Training)

1 **begin initialize** $j \leftarrow 0$, $n \leftarrow$ # patterns, $\epsilon \leftarrow$ small param, $\lambda_m \leftarrow$ max radius
2 **do** $j \leftarrow j + 1$
3 $w_{ij} \leftarrow x_i$ (train weight)
4 $\hat{\mathbf{x}} \leftarrow \arg\min_{\mathbf{x} \notin \omega_i} D(\mathbf{x}, \mathbf{x}')$ (find nearest point not in ω_i)
5 $\lambda_j \leftarrow \min[D(\hat{\mathbf{x}}, \mathbf{x}') - \epsilon, \lambda_m]$ (set radius)
6 **if** $\mathbf{x} \in \omega_k$ **then** $a_{jk} \leftarrow 1$
7 **until** $j = n$
8 **end**

Reduced Coulomb Energy Network

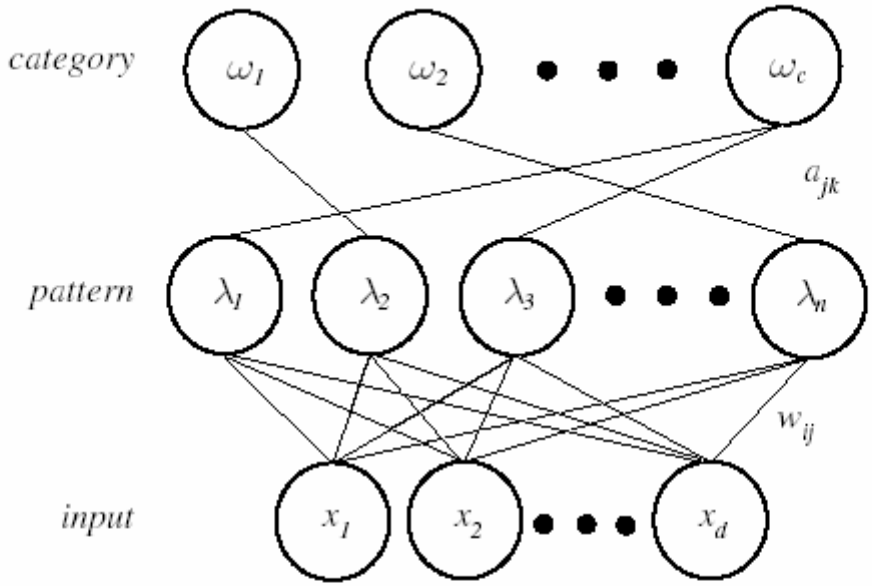

FIGURE 4.25. An RCE network is topologically equivalent to the PNN of Fig. 4.9. During training, normalized weights are adjusted to have the same values as the normalized pattern presented, just as in a PNN. In this way, distances can be calculated by an inner product. Pattern units in an RCE network also have a modifiable threshold corresponding to a "radius" λ. During training, each threshold is adjusted so that its radius is as large as possible without containing training patterns from a different category. From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001 by John Wiley & Sons, Inc.

Classification with Reduced Coulomb Energy Networks

Algorithm 5. (RCE Classification)

1 **begin initialize** $j \leftarrow 0, k \leftarrow 0, \mathbf{x} \leftarrow$ test pattern, $\mathcal{D}_t \leftarrow \{\}$
2 **do** $j \leftarrow j + 1$
3 **if** $D(\mathbf{x}, \mathbf{x}'_j) < \lambda_j$ **then** $\mathcal{D}_t \leftarrow \mathcal{D}_t \cup \mathbf{x}'_j$
4 **until** $j = n$
5 **if** label of all $\mathbf{x}'_j \in \mathcal{D}_t$ is the same **then return** label of all $\mathbf{x}_k \in \mathcal{D}_t$
6 **else return** "ambiguous" label
7 **end**